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General Certificate of Education (Advanced Level) – Grade 12

UNIT TEST

Combined Mathematics I

Coverage: Real numbers & surds · Functions · Quadratic functions & equations · Polynomials · Rational functions & partial fractions · Index & logarithmic laws · Angular measure · Cartesian coordinates · Circular functions · Trigonometric identities · Sine & cosine rules (statement) · Inequalities (incl. modulus)

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Time: 5 hours
Part A – 2 h 30 min · Part B – 2 h 30 min
(may be taken in two sittings)

Index No. | | | | | | | |

Name : .....

School : .....

Instructions

- This paper consists of two parts: **Part A** (Questions 1–30) and **Part B** (Questions 31–46).
- **Part A:** Answer **all** questions. Write your answers **in the space provided on this paper**. Additional sheets may be used if needed.
- **Part B:** Answer **five** questions only. Write your answers **in the answer book provided**.
- Part A carries 25 marks per question (750). Part B carries 150 marks per question (750). Total 1500.
- Questions in each part become progressively more demanding.
- At the end of the allotted time, tie Part A on top of your Part B answer book and hand both to the supervisor.
- Show all working. Marks are awarded for correct method as well as correct answers.

For Examiners' use only

Part	Q	Mk	Q	Mk	Q	Mk
A	1		11		21	
	2		12		22	
	3		13		23	
	4		14		24	
	5		15		25	
	6		16		26	
	7		17		27	
	8		18		28	
	9		19		29	
	10		20		30	
B	31		37		43	
	32		38		44	
	33		39		45	
	34		40		46	
	35		41			
	36		42			
				Total		

Total	
Part A (750)	
Part B (750)	
In numbers (1500)	
In words	

Code numbers	
Marking examiner	
Checked by 1	
2	
Supervised by	

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PART A

Answer **all** questions in the space provided.

1. Classify each number as natural, integer, rational or irrational: -7, 0.375, $\sqrt{49}$, $\sqrt{50}$, π. Express $0.\dot{3}\dot{6}$ and $1.2\dot{5}$ as fractions in their lowest terms. **(25 marks)**

2. Simplify $\sqrt{75} - 2\sqrt{12} + \sqrt{27}$. Rationalise the denominator of $\dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$. **(25 marks)**

3. Expand $(2x - 3y)^3$. Factorise $8a^3 - 27b^3$, and simplify $\dfrac{2}{x^2-1} - \dfrac{1}{x^2+x}$. **(25 marks)**

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4. Convert 225° to radians and $\frac{7\pi}{6}$ to degrees. A sector of a circle has radius 6 cm and angle $\frac{2\pi}{3}$ at the centre. Find its arc length and its area. **(25 marks)**

5. Find the distance between $(-2, 3)$ and $(4, -5)$. For $A(1, -2)$ and $B(7, 4)$, find the coordinates of the point dividing AB in the ratio $1 : 2$ (i) internally, (ii) externally. **(25 marks)**

6. Write down the exact values of $\sin\frac{5\pi}{6}$, $\cos\frac{4\pi}{3}$, $\tan\frac{7\pi}{4}$ and $\sec\left(-\frac{\pi}{3}\right)$. Given $\cos\theta = -\frac{3}{5}$ and $\frac{\pi}{2} < \theta < \pi$, find $\sin\theta$ and $\tan\theta$. **(25 marks)**

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7. In the usual notation for a triangle ABC, state the sine rule and the cosine rule. Using them, find (i) c when $a = 7$, $b = 5$, $C = 60°$; (ii) b when $A = 45°$, $B = 60°$, $a = 10$. **(25 marks)**

8. Let $f(x) = 3x - 2$ and $g(x) = x^2 + 1$ for $x \in \mathbb{R}$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$. State the largest subset of $\mathbb{R}$ that can be the domain of (i) $\sqrt{5 - x}$, (ii) $\dfrac{1}{\sqrt{x^2 - 9}}$. **(25 marks)**

9. Sketch the graph of $f(x) = \begin{cases} x + 1, & x < 0 \\ 1, & 0 \leq x \leq 2 \\ 3 - x, & x > 2 \end{cases}$ and state the range of f. **(25 marks)**

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10. Evaluate $\dfrac{8^{2/3} \times 16^{-1/4}}{27^{-1/3}}$. Solve $9^x = 27^{x-1}$. Express $\log \dfrac{x^2y^3}{\sqrt{z}}$ in terms of $\log x$, $\log y$ and $\log z$. **(25 marks)**

11. Solve the simultaneous equations $x + y + z = 6, \quad 2x - y + z = 3, \quad x + 2y - z = 2$. **(25 marks)**

12. Find the quotient and the remainder when $2x^3 - 3x^2 + 4x - 5$ is divided by $x - 2$. Show that $x - 3$ is a factor of $x^3 - 2x^2 - 5x + 6$ and factorise it completely. **(25 marks)**

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13. Express $2x^2-12x+7$ in the form $a(x+b)^2+c$ and state its least value. The roots of $2x^2-5x+1=0$ are α and β. Find $\alpha^2+\beta^2$ and $(\alpha-\beta)^2$. **(25 marks)**

14. Express in partial fractions: (i) $\dfrac{x+5}{x^2+x-2}$, (ii) $\dfrac{3x^2+2x+2}{(x-2)(x+1)^2}$. **(25 marks)**

15. Find the set of real values of x satisfying (i) $x^2-5x+6>0$, (ii) $\dfrac{x-1}{2x+1}>1$. **(25 marks)**

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16. Find the set of real values of x satisfying (i) $|2x+1| \geq 5$, (ii) $|x+1| > 3x+7$. **(25 marks)**

17. A sector of radius 5 cm has perimeter 20 cm. Find its angle and area. A chord of a circle of radius 10 cm subtends an angle $\frac{\pi}{3}$ at the centre. Find the area of the minor segment. **(25 marks)**

18. Show that $A(1,1)$, $B(4,5)$ and $C(8,2)$ are the vertices of an isosceles right-angled triangle. Find the equation of the locus of a point which is equidistant from $(1,2)$ and $(3,-4)$. **(25 marks)**

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19. Simplify $\dfrac{\sin(\pi+\theta)\cos\left(\frac{\pi}{2}-\theta\right)\tan\left(\frac{3\pi}{2}+\theta\right)}{\cos(2\pi-\theta)\sin\left(\frac{\pi}{2}+\theta\right)\cot(\pi-\theta)}$. **(25 marks)**

20. Using $75^\circ = 45^\circ + 30^\circ$, find the exact values of $\sin 75^\circ$ and $\tan 75^\circ$. Prove that $(1+\tan\theta)^2 + (1+\cot\theta)^2 = (\sec\theta + \operatorname{cosec}\theta)^2$. **(25 marks)**

21. Let $f(x) = \dfrac{2x+1}{x-3}$ for $x \in \mathbb{R} \setminus \{3\}$. Find the range of f, show that f is one-one, and find $f^{-1}(x)$. **(25 marks)**

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22. The roots of $x^2 - 3x + 1 = 0$ are α and β. Without solving the equation, find the quadratic equations whose roots are (i) $\alpha + 2$ and $\beta + 2$, (ii) α^2 and β^2. **(25 marks)**

23. Find the set of values of k for which $(k-2)x^2 + 2(k-2)x + 3 > 0$ for all real x. Find also the non-zero value of k for which $x^2 - 5x + k = 0$ and $x^2 - 7x + 2k = 0$ have a common root. **(25 marks)**

24. $p(x) = x^3 + ax^2 + bx - 6$. Given that $(x-1)$ is a factor of $p(x)$ and that the remainder when $p(x)$ is divided by $(x+1)$ is -4, find a and b and express $p(x)$ as a product of linear factors. Find the remainder when $p(x)$ is divided by $x^2 - 1$. **(25 marks)**

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25. Solve (i) $3^{2x+1} - 28 \cdot 3^x + 9 = 0$, (ii) $\log_2 x + \log_x 2 = \frac{5}{2}$. **(25 marks)**

26. Find the set of real values of x satisfying (i) $\frac{x+2}{5-2x} \geq 4$, (ii) $\frac{1}{x+2} \leq \frac{x}{3}$. **(25 marks)**

27. Given that $\sin\theta = -\frac{1}{3}$ and $\pi < \theta < \frac{3\pi}{2}$, show that $\sin 2\theta = \frac{4\sqrt{2}}{9}$ and $\tan 2\theta = \frac{4\sqrt{2}}{7}$. Find the exact value of $\tan\frac{\theta}{2}$. **(25 marks)**

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28. Let $f(x) = 2x^2 - 8x + 5$ with domain $[k, \infty)$. Find the least value of k for which f is one-one. For this value of k, find the range of f and $f^{-1}(x)$. **(25 marks)**

29. Sketch the graphs of $y = |x| + 1$ and $y = 2|x - 1|$ in the same diagram, and hence find all real x satisfying $|x| + 1 < 2|x - 1|$. **(25 marks)**

30. Sketch the graphs of $y = |2x - 3|$ and $y = 3 - |x|$ in the same diagram. Hence find all real x satisfying $|2x - 3| + |x| < 3$, and deduce all real x satisfying $|4x - 3| + 2|x| < 3$. **(25 marks)**

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PART B

Answer **five** questions only, in the answer book provided.

31. **(150 marks)**

(a) Find rational numbers a, b such that $\sqrt{7+4\sqrt{3}} = a + b\sqrt{3}$. Show that $\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} = 1$.

(b) If $a = b + c$, show that $a^3 - b^3 - c^3 = 3abc$. Hence factorise $27(x+2)^3 - (2x+3)^3 - (x+3)^3$.

(c) Solve the simultaneous equations $x + y = 5$ and $x^2 + xy + y^2 = 19$.

(d) Prove that $\sqrt{2}$ is irrational, and deduce that $3 + 2\sqrt{2}$ is irrational.

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32. **(150 marks)**

(a) Let $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$. Show that f is neither one-one nor onto. Show that, with domain and codomain both $[0, \infty)$, f has an inverse, and find it.

(b) Let $f(x) = \frac{1+x}{1-x}$, $x \neq 1$. Show that $(f \circ f)(x) = -\frac{1}{x}$ for $x \neq 0, 1$, and deduce that $(f \circ f \circ f \circ f)(x) = x$.

(c) Let $h : \mathbb{R} \to \mathbb{R}$, $h(x) = x|x|$. Sketch $y = h(x)$, show that h is one-one and onto, and find $h^{-1}(x)$.

(d) Find all linear functions $g(x) = ax + b$ satisfying $g(g(x)) = 4x - 9$ for all x. Find also the function f satisfying $f(x) + 2f\left(\frac{1}{x}\right) = 3x$ for all $x \neq 0$.

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33. **(150 marks)**

(a) The roots of $x^2 + px + q = 0$ are α and β, where $q \neq 0$. Show that the quadratic equation whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$ is $qx^2 + p(q+1)x + (q+1)^2 = 0$. Show that these roots are real if and only if $p^2 \geq 4q$ or $q = -1$.

(b) Let $f(x) = 2x^2 - 4x + 5$ and $g(x) = -x^2 + 2kx - 3$. By completing the square, find the least value of $f(x)$ and the greatest value of $g(x)$.

(i) Find the range of k for which the least value of f exceeds the greatest value of g.

(ii) Find the range of k for which the graphs $y = f(x)$ and $y = g(x)$ do not meet.

(iii) Explain why every k in (i) lies in (ii), but not conversely.

(c) Find the range of $y = \frac{x^2 + 2x + 3}{x^2 + 1}$ for real x.

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34. **(150 marks)**

(a) Show that the roots of $(x-a)(x-b) = c^2$ are real for all real a, b, c, and state when they are equal.

(b) Let $k, \lambda \in \mathbb{R}$, $k \neq 0$. Show that $kx^2 - 2\lambda x - \frac{1}{k} = 0$ has two distinct real roots. If these roots are α, β, show that $\alpha^2 + \beta^2 = \frac{4\lambda^2 + 2}{k^2}$.

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(c) The roots of $ax^2 + bx + c = 0$ are α, β. Show that the equation whose roots are $\dfrac{\alpha - 1}{\alpha + 1}$ and $\dfrac{\beta - 1}{\beta + 1}$ is $(a - b + c)x^2 + 2(a - c)x + (a + b + c) = 0$.

(d) α, β are the roots of $x^2 + px + q = 0$ and γ, δ are the roots of $x^2 + rx + s = 0$. Show that $(\alpha - \gamma)(\beta - \gamma)(\alpha - \delta)(\beta - \delta) = (q - s)^2 + (p - r)(ps - qr)$. Deduce the condition for the two equations to have a common root.

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35. **(150 marks)**

(a) State and prove the remainder theorem. Show that when a polynomial $p(x)$ is divided by $(x - a)(x - b)$, $a \neq b$, the remainder is $\dfrac{p(a) - p(b)}{a - b}x + \dfrac{a\,p(b) - b\,p(a)}{a - b}$.

(b) When $f(x)$ is divided by $(x - 1)(x - 2)(x - 3)$ the remainder is $\lambda(x - 2)(x - 3) + \mu(x - 3)(x - 1) + \nu(x - 1)(x - 2)$. Find $f(1)$, $f(2)$ and $f(3)$ in terms of λ, μ, ν. When $x^5 + kx^2$ is divided by $(x - 1)(x - 2)(x - 3)$, the remainder has no term in x^2. Find k.

(c) Let $f(x) = x^4 + ax^3 + px^2 + q$ and $g(x) = x^4 + bx^3 - qx^2 + 2x - p$. Given that $(x + 1)$ is a factor of $f(x)$ and that $g(x)$ leaves remainder 4 on division by $(x - 1)$, show that $a = b$. Given further that $f(x) = f(-x)$ has a non-zero real solution, show that $a = 0$ and $p + q = -1$, and hence that $(x + 1)$ is a factor of $g(x)$.

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36. **(150 marks)**

(a) Find the constants A, B, C such that $3x^2 + 5x - 4 \equiv A(x + 1)^2 + B(x + 1) + C$.

(b) Let $p(x) = 2x^3 + ax^2 + bx - 6$. Given that $(x - 2)$ is a factor of $p(x)$ and that the remainder on division by $(x + 1)$ is -12, find a and b. Factorise $p(x)$ and show that $p(x) = 0$ has exactly one real root.

(c) By putting $y = 7 - x$, solve $(6 - x)^4 + (8 - x)^4 = 16$.

(d) Let $n \geq 2$ be an integer and $h(x) = x^n - nx + (n - 1)$. Show that $(x - 1)$ is a factor of $h(x)$. By writing $h(x) = (x^n - 1) - n(x - 1)$, show that $(x - 1)^2$ is a factor of $h(x)$.

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37. **(150 marks)**

(a) By dividing by x^2 and putting $t = x + \frac{1}{x}$, solve $2x^4 + x^3 - 11x^2 + x + 2 = 0$.

(b) Solve $(x + 2)(x + 3)(x + 8)(x + 12) = 4x^2$.

(c) Express $\dfrac{6t^2 + 7t + 3}{(2t - 1)(4t^2 + 4t + 5)}$ in partial fractions.

(d) Express $\dfrac{2x^3 + x^2 - x + 3}{x^2(x + 1)}$ in partial fractions.

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38. **(150 marks)**

(a) Express $\dfrac{1}{(x^2 - 1)(x^2 + 4)}$ in partial fractions.

(b) Express $\dfrac{x^4 + 1}{x^2 - 1}$ in the form $Q(x) + \dfrac{A}{x - 1} + \dfrac{B}{x + 1}$, where $Q(x)$ is a polynomial.

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(c) Express $\dfrac{1}{r(r+1)(r+2)}$ in partial fractions, and show that it equals $\dfrac{1}{2}\left[\dfrac{1}{r(r+1)} - \dfrac{1}{(r+1)(r+2)}\right]$.

(d) Find p and q such that, in the partial fractions of $\dfrac{x^2+px+q}{(x-1)(x-2)(x-3)}$, there is no term in $\frac{1}{x-2}$ and the coefficient of $\frac{1}{x-1}$ is 2. Write down the partial fractions.

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39. **(150 marks)**

(a) Prove that $\log_a b = \dfrac{\log_c b}{\log_c a}$. Show that $\dfrac{1}{\log_2 N} + \dfrac{1}{\log_3 N} + \dfrac{1}{\log_4 N} = \dfrac{1}{\log_{24} N}$.

(b) Show that $\log_9(xy^2) = \frac{1}{2}\log_3 x + \log_3 y$. Hence solve $\log_9(xy^2) = \frac{1}{2}$, $\log_3 x \cdot \log_3 y = -3$.

(c) Solve $\log_8 x + \log_4 y = \frac{5}{2}$ and $\log_8 y + \log_4 x = \frac{5}{3}$ simultaneously.

(d) Solve $64 \cdot 9^x - 84 \cdot 12^x + 27 \cdot 16^x = 0$.

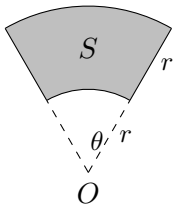
(e) If $a, b > 0$ and $a^2 + b^2 = 7ab$, show that $\log \dfrac{a+b}{3} = \frac{1}{2}(\log a + \log b)$.

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40. **(150 marks)**

(a) A sector of a circle has perimeter 40 cm and radius r cm. Show that its area is $r(20-r)$ cm^2. By completing the square, find the greatest area and the angle of the sector in that case.

(b) The shaded region S in the figure is bounded by two arcs of radii r and $2r$ with centre O, both subtending an angle θ at O, and by two straight edges. The area of S is 81 m^2.



Show that the perimeter of S is $p = 2\left(r + \dfrac{81}{r}\right)$. Prove that $r + \dfrac{81}{r} \geq 18$ for $r > 0$, and hence find the least value of p and the corresponding values of r and θ.

(c) Sketch $y = 2\sin\left(x - \frac{\pi}{6}\right) + 1$ for $0 \leq x \leq 2\pi$, stating its greatest and least values and where they occur.

(d) By sketching $y = \sin x$ and $y = \dfrac{x}{\pi}$ on the same diagram, show that the equation $\sin x = \dfrac{x}{\pi}$ has exactly three real solutions.

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41. **(150 marks)**

(a) Prove that $\tan 3\theta \tan 2\theta \tan\theta = \tan 3\theta - \tan 2\theta - \tan\theta$, and that $\dfrac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta} = \tan\dfrac{\theta}{2}$.

(b) Show that $4\sin\theta\sin(60^\circ-\theta)\sin(60^\circ+\theta) = \sin 3\theta$, and hence that $\sin 20^\circ \sin 40^\circ \sin 80^\circ = \dfrac{\sqrt{3}}{8}$.

(c) By expressing $\sin 3\theta$ and $\cos 2\theta$ in terms of $s = \sin\theta$, show that $s = \sin 18^\circ$ satisfies $4s^3 - 2s^2 - 3s + 1 = 0$. Factorise this cubic and show that $\sin 18^\circ = \dfrac{\sqrt{5}-1}{4}$. Deduce the exact value of $\cos 36^\circ$.

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(d) Express $\sqrt{3}\cos x - \sin x$ in the form $R\cos(x+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Find the greatest and least values of $\dfrac{1}{3+\sqrt{3}\cos x - \sin x}$.

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42. **(150 marks)**

(a) Show that $8\cos^4\theta - 4\cos^3\theta - 8\cos^2\theta + 3\cos\theta + 1 = \cos 4\theta - \cos 3\theta$.

(b) Find constants a and b such that $\cos^6\theta + \sin^6\theta = a + b\cos 4\theta$ for all θ.

(c) Show that $\tan x + \tan\left(\frac{\pi}{3} + x\right) + \tan\left(\frac{2\pi}{3} + x\right) = 3\tan 3x$.

(d) By multiplying by $2\sin\frac{\pi}{7}$, show that $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} = -\frac{1}{2}$.

(e) If $A + B + C = \pi$, prove that $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$.

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43. **(150 marks)**

(a) Prove that $\dfrac{\sin x + \sin 3x + \sin 5x + \sin 7x}{\cos x + \cos 3x + \cos 5x + \cos 7x} = \tan 4x$ and $\sec\theta + \tan\theta = \tan\left(\dfrac{\pi}{4} + \dfrac{\theta}{2}\right)$.

(b) Show that $\sin 22\frac{1}{2}^\circ = \frac{1}{2}\sqrt{2-\sqrt{2}}$ and $\tan 22\frac{1}{2}^\circ = \sqrt{2} - 1$. Find the exact value of $\tan 67\frac{1}{2}^\circ$.

(c) Prove that $\dfrac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta} = \tan\theta$.

(d) If $A + B + C = \pi$, prove that $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$. In a triangle ABC with $a : b : c = 1 : \lambda : \mu$, use the sine rule to show that $\mu^2(\sin 2A + \sin 2B + \sin 2C) = 4\lambda\sin^3 C$.

(e) Sketch $y = \sin x$ and $y = \cos 2x$ for $0 \le x \le 2\pi$ on the same diagram, and state the number of points of intersection.

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44. **(150 marks)**

(a) For real a, b, c, prove that $a^2 + b^2 + c^2 \ge ab + bc + ca$. Show that $a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a+b+c)\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$, and deduce that for positive a, b, c: (i) $a^3 + b^3 + c^3 \ge 3abc$, (ii) $\dfrac{a+b+c}{3} \ge \sqrt[3]{abc}$, (iii) $(a+b+c)\left(\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}\right) \ge 9$.

(b) Find the set of real x satisfying $\dfrac{2x}{2x^2 + 5x + 2} \ge \dfrac{1}{x+1}$.

(c) Show that $\dfrac{1}{3} \le \dfrac{x^2 - x + 1}{x^2 + x + 1} \le 3$ for all real x.

(d) Solve $\dfrac{6(3x+1)}{(x-1)(x-2)(x+3)} < 1$.

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45. **(150 marks)**

(a) For $a, b > 0$, prove that (i) $\dfrac{a}{b} + \dfrac{b}{a} \ge 2$, (ii) $\dfrac{1}{a} + \dfrac{1}{b} \ge \dfrac{4}{a+b}$.

(b) For $a, b, c, d > 0$, show that $(a+b+c+d)\left(\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c} + \dfrac{1}{d}\right) \ge 16$.

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(c) If $a > 1$ and $b > 1$, show that $2(ab + 1) > (a + 1)(b + 1)$. Deduce that if $a, b, c, d > 1$ then $8(abcd + 1) > (a + 1)(b + 1)(c + 1)(d + 1)$.

(d) Find the set of real x satisfying $\dfrac{x^2 + 63}{x} \geq 16$.

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46. **(150 marks)**

(a) Sketch the graphs of $y = |2x - 3| - 1$ and $y = 5 - |3x|$ in the same diagram. Hence, or otherwise, find all real x satisfying $|2x-3|+|3x| < 6$, and deduce the set of real x satisfying $|4x-3|+6|x| < 6$.

(b) Sketch $y = 2|x + 1| - 3$ and $y = 2|x - 1| + x$ in the same diagram. Hence, or otherwise, solve the inequality $x + 2|x - 1| > 2|x + 1| - 3$ and the equation $x + 2|x - 1| = 2|x + 1| - 3$.

(c) Solve $3|x| \geq |6x - 3| - 5$. By sketching $y = 3|x| - |6x - 3|$, find the value of α for which $3|x| = |6x - 3| + \alpha$ has exactly one real solution.

(d) The set of all real x satisfying $|3x - a| < |bx - 2|$ is $\{x : x > \frac{4}{3}\}$, where $a, b \in \mathbb{R}$. Find all possible pairs (a, b).

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