G.C.E. (A/L) Grade 12 – Unit Test

Answer Key – Final Answers

Combined Mathematics I and II · "Show that" results are not repeated; full marks need full working.

Combined Mathematics I

Part A

1. $-7 \in \mathbb{Z}$; $0.375 \in \mathbb{Q}$; $\sqrt{49} = 7 \in \mathbb{N}$; $\sqrt{50}, \pi$ irrational. $0.\dot{3}\dot{6} = \frac{4}{11}$, $1.2\dot{5} = \frac{113}{90}$.
2. $4\sqrt{3}$; $\frac{7+2\sqrt{10}}{3}$.
3. $8x^3 - 36x^2y + 54xy^2 - 27y^3$; $(2a-3b)(4a^2+6ab+9b^2)$; $\frac{1}{x(x-1)}$.
4. $\frac{5\pi}{4}$; $210°$; arc 4π cm, area 12π cm^2.
5. 10; (i) $(3,0)$ (ii) $(-5,-8)$.
6. $\frac{1}{2}, -\frac{1}{2}, -1, 2$; $\sin\theta = \frac{4}{5}$, $\tan\theta = -\frac{4}{3}$.
7. (i) $\sqrt{39}$ (ii) $5\sqrt{6}$.
8. $3x^2+1$; $9x^2-12x+5$; (i) $x \le 5$ (ii) $|x| > 3$.
9. Range $(-\infty, 1]$.
10. 6; $x = 3$; $2\log x + 3\log y - \frac{1}{2}\log z$.
11. $x = 1,\ y = 2,\ z = 3$.
12. $2x^2 + x + 6$, remainder 7; $(x-3)(x+2)(x-1)$.
13. $2(x-3)^2 - 11$, least value -11 at $x = 3$; $\frac{21}{4}$, $\frac{17}{4}$.
14. (i) $\frac{2}{x-1} - \frac{1}{x+2}$ (ii) $\frac{2}{x-2} + \frac{1}{x+1} - \frac{1}{(x+1)^2}$.
15. (i) $x < 2$ or $x > 3$ (ii) $-2 < x < -\frac{1}{2}$.
16. (i) $x \le -3$ or $x \ge 2$ (ii) $x < -2$.
17. 2 rad, 25 cm^2; $50\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)$ cm^2.
18. $AB = BC = 5$, $AC = \sqrt{50}$; locus $x - 3y = 5$.
19. $-\tan^2\theta$.
20. $\sin 75° = \frac{\sqrt{6}+\sqrt{2}}{4}$, $\tan 75° = 2 + \sqrt{3}$.
21. Range $\mathbb{R} \setminus \{2\}$; $f^{-1}(x) = \frac{3x+1}{x-2}$.
22. (i) $x^2 - 7x + 11 = 0$ (ii) $x^2 - 7x + 1 = 0$.
23. $2 \le k < 5$; $k = 6$ (common root 3).
24. $a = 4, b = 1$; $(x-1)(x+2)(x+3)$; remainder $2x - 2$.
25. (i) $x = -1$ or 2 (ii) $x = 4$ or $\sqrt{2}$.
26. (i) $2 \le x < \frac{5}{2}$ (ii) $-3 \le x < -2$ or $x \ge 1$.
27. $\tan\frac{\theta}{2} = -(3 + 2\sqrt{2})$.
28. $k = 2$; range $[-3, \infty)$; $f^{-1}(x) = 2 + \sqrt{\frac{x+3}{2}}$.
29. $x < \frac{1}{3}$ or $x > 3$.
30. $0 < x < 2$; deduced $0 < x < 1$.

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Part B

31. (a) $a = 2, b = 1$. (b) $9(x+2)(2x+3)(x+3)$. (c) $(2,3), (3,2)$.
32. (a) $f^{-1}(x) = \sqrt{x}$. (c) $h^{-1}(x) = \sqrt{x}$ $(x \geq 0)$, $-\sqrt{-x}$ $(x < 0)$. (d) $g(x) = 2x - 3$ or $-2x + 9$; $f(x) = \frac{2}{x} - x$.
33. (b) least $f = 3$; greatest $g = k^2 - 3$. (i) $-\sqrt{6} < k < \sqrt{6}$ (ii) $-2 - 2\sqrt{6} < k < -2 + 2\sqrt{6}$. (c) $2 - \sqrt{2} \leq y \leq 2 + \sqrt{2}$.
34. (a) equal iff $a = b$ and $c = 0$. (d) common root iff $(q-s)^2 + (p-r)(ps-qr) = 0$.
35. (b) $f(1) = 2\lambda,\ f(2) = -\mu,\ f(3) = 2\nu$; $k = -90$.
36. (a) $A = 3, B = -1, C = -6$. (b) $a = -3, b = 1$; $(x-2)(2x^2 + x + 3)$. (c) $x = 6$ or 8.
37. (a) $2,\ \frac{1}{2},\ \frac{-3 \pm \sqrt{5}}{2}$. (b) $-4,\ -6,\ \frac{-15 \pm \sqrt{129}}{2}$. (c) $\frac{1}{2t-1} + \frac{t+2}{4t^2+4t+5}$. (d) $2 - \frac{4}{x} + \frac{3}{x^2} + \frac{3}{x+1}$.
38. (a) $\frac{1}{10(x-1)} - \frac{1}{10(x+1)} - \frac{1}{5(x^2+4)}$. (b) $x^2 + 1 + \frac{1}{x-1} - \frac{1}{x+1}$. (c) $\frac{1}{2r} - \frac{1}{r+1} + \frac{1}{2(r+2)}$. (d) $p = -7, q = 10$; $\frac{2}{x-1} - \frac{1}{x-3}$.
39. (b) $x = 27, y = \frac{1}{3}$ or $x = \frac{1}{9}, y = 3\sqrt{3}$. (c) $x = 1, y = 32$. (d) $x = 1$ or 2.
40. (a) 100 cm^2, angle 2 rad. (b) $p_{\min} = 36$ m at $r = 9$, $\theta = \frac{2}{3}$. (c) greatest 3 at $\frac{2\pi}{3}$; least -1 at $\frac{5\pi}{3}$.
41. (c) $\cos 36^\circ = \frac{1+\sqrt{5}}{4}$. (d) $2\cos\left(x + \frac{\pi}{6}\right)$; greatest 1, least $\frac{1}{5}$.
42. (b) $a = \frac{5}{8},\ b = \frac{3}{8}$.
43. (b) $\tan 67\frac{1}{2}^\circ = \sqrt{2} + 1$. (e) 3 points.
44. (b) $-2 < x < -1$ or $-\frac{2}{3} \leq x < -\frac{1}{2}$. (d) $x < -5$, $-3 < x < 0$, $1 < x < 2$ or $x > 5$.
45. (d) $0 < x \leq 7$ or $x \geq 9$.
46. (a) $-\frac{3}{5} < x < \frac{9}{5}$; deduced $-\frac{3}{10} < x < \frac{9}{10}$. (b) $x > -7,\ x \neq 1$; equation: $x = -7$ or 1. (c) $-\frac{2}{3} \leq x \leq \frac{8}{3}$; $\alpha = \frac{3}{2}$. (d) $(6,3)$ or $(10,-3)$.

Combined Mathematics II

Part A

1. (i) $\overrightarrow{AD}$ (ii) $\overrightarrow{AC}$ (iii) $\mathbf{0}$.
2. 5; $\frac{1}{5}(3\mathbf{i} + 4\mathbf{j})$; $6\mathbf{i} + 8\mathbf{j}$; parallel, $-3\mathbf{i} + 9\mathbf{j} = -\frac{3}{2}(2\mathbf{i} - 6\mathbf{j})$.
3. $\mathbf{i} - 11\mathbf{j}$, $\sqrt{122}$; $t = \frac{1}{3}$.
4. $5\sqrt{3}\,\mathbf{i} + 5\mathbf{j}$; $25\sqrt{3}$ N, 25 N.
5. (i) 5 N (ii) $\sqrt{129}$ N.
6. 120°.
7. $3\mathbf{i} + 4\mathbf{j}$, 5; $\frac{1}{5}(3\mathbf{p} + 2\mathbf{q})$.
8. $\pm\frac{1}{13}(5\mathbf{i} - 12\mathbf{j})$; $-10\mathbf{i} + 24\mathbf{j}$.
9. $\lambda = 3, \mu = -2$.
10. 5; $\frac{5}{\sqrt{221}}$; $p = \frac{8}{3}$.
11. $1, 0, \mathbf{k}, -\mathbf{k}, \mathbf{0}$; $-5\mathbf{k}$.
12. $4\mathbf{i} + 5\mathbf{j}$, $\sqrt{41}$ N; $2\mathbf{i} - 2\mathbf{j}$.
13. 10 N at $\tan^{-1}\frac{4}{3}$ south of east.
14. —
15. $\lambda = \frac{1}{2},\ \mu = \frac{5}{2}$.

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16. $\overrightarrow{AB} = -3\overrightarrow{BC}$; $AC : CB = 2 : 1$.
17. $\mathbf{b} - \mathbf{a}$, $-\mathbf{a}$, $\mathbf{a} + \mathbf{b}$, $2\mathbf{b}$, $2\mathbf{b} - \mathbf{a}$.
18. $\frac{7}{\sqrt{2}}$; $\pm\frac{1}{5}(4\mathbf{i} - 3\mathbf{j})$.
19. $90°$; $\sqrt{13}$.
20. $(\mathbf{i} + \mathbf{j}) \times (\mathbf{i} - \mathbf{j}) = -2\mathbf{k} = 2(\mathbf{j} \times \mathbf{i})$.
21. 12 N and 5 N; 13 N; $60°$.
22. $\sqrt{3}\,P$.
23. $\sqrt{3}\,P$ at $210°$ to Ox.
24. $135°$; $\lambda = \frac{1}{2}$.
25. $60°$; $\sqrt{19}$; $\lambda = -\frac{4}{3}$.
26. —
27. —
28. $Q = \sqrt{2}\,P$; $135°$.
29. $P = 9,\ Q = 6$.
30. $\overrightarrow{OP} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{c}$.

Part B

31. (a) $\mathbf{b} - \mathbf{a}$, $-\mathbf{a}$, $-\mathbf{b}$, $\mathbf{a} - \mathbf{b}$, $\mathbf{a} + \mathbf{b}$, $2\mathbf{b}$, $2\mathbf{b} - \mathbf{a}$, $\mathbf{b} - \mathbf{a}$.
32. (b) $\lambda = -1$.
33. (a) $\overrightarrow{AB} = 3\mathbf{i} + 4\mathbf{j}$, $\overrightarrow{BC} = 4\mathbf{i} - 3\mathbf{j}$; area $\frac{25}{2}$; $D = 6\mathbf{i} - 2\mathbf{j}$. (b) $\frac{1}{5\sqrt{2}}$; $\lambda = \frac{1}{10}$, $\mathbf{c} = \frac{7}{10}(\mathbf{i} + 3\mathbf{j})$.
34. (i) $\overrightarrow{OM} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$, $\overrightarrow{CM} = -\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$; (iv) $\overrightarrow{MP} = \frac{1}{4}\mathbf{b}$.
35. (a) $\lambda = \frac{4}{5},\ \mu = \frac{2}{5}$; $OF : FD = 4 : 1$, $AF : FE = 2 : 3$. (b) $AF : FC = 1 : 3$.
36. (a) $\overrightarrow{OR} = \frac{1}{9}\mathbf{a} + \frac{2}{3}\mathbf{b}$; $AR : RQ = 8 : 1$; $BR : RP = 1 : 2$.
37. (a) $AC : CB = 1 : 3$.
38. —
39. (a) $20(\sqrt{3} - 1)$ N along the $30°$ line, $10(\sqrt{6} - \sqrt{2})$ N along the $45°$ line. (b) $\sqrt{31}$ N at $\approx 128.9°$ to Ox. (c) $\sqrt{3}$ N. (d) $-2\mathbf{i} - \mathbf{j}$ ($\sqrt{5}$ N).
40. (a) $P = 3, Q = 5$ or $P = -1, Q = 1$. (b) $60°$. (c) $-\frac{1}{8}$.
41. —
42. (a) $\cos\alpha = \frac{3}{5}$.
43. (c) $\sqrt{3}\,a$.
44. (a) $T = R = \frac{W}{\sqrt{3}}$. (b) $P = W\tan\alpha$, $R = W\sec\alpha$. (c) $\frac{4W}{5}$ ($3a$ string), $\frac{3W}{5}$ ($4a$ string).
45. (a)(i) $W\tan\theta$, $T = W\sec\theta$ (ii) $W\sin\theta$, $T = W\cos\theta$. (b) $\frac{Wl}{2\sqrt{l^2 - d^2}}$.
46. (a) $R = 10\sqrt{3} - 10\tan\theta$; contact for $0 \le \theta \le 60°$. (b) 17 N, opposite to the resultant of the two forces, i.e. at $180° + \tan^{-1}\frac{15}{8}$ to the 8 N force.

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